

# Economic Parameters in the Conceptual Design Optimization of an Air-Taxi Aircraft

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A new methodology for the conceptual design optimization of a public transport vehicle for a specific network is proposed, in which the importance that the passengers assign to their time is incorporated as a parameter called value of time  $\mathcal{V}$ . A penalty function  $\mathcal{P}$  is developed, which is a combination of the fare, the time spent in travel, and a comfort factor. For operation over virgin areas, the sizing of the transport termini and estimation of the costs associated with their construction is an intrinsic part of the vehicle optimization studies. As an example, this methodology is integrated in the conceptual design of twin-engined pistonprop and turboprop general aviation aircraft to be employed for air-taxi operations over a hypothetical network. The optimum location of the airport from the city center, and the minimum runway length required for safe operation is determined. The costs associated with construction and maintenance of a runway at each node of the network  $C_{rw}$  are estimated and charged to the passengers as a part of their fare. A set of eight aircraft related parameters (design variables) that correspond to the minimum  $\mathcal{P}$  is obtained, while assigning nominal values to the remaining parameters. Eight test-cases are investigated and the sensitivity of  $\mathcal{P}$  to the design variables is determined. It is found that the aircraft capacity and the cruising speed are directly dependent on the  $\mathcal{V}$ , and the inclusion of  $C_{rw}$  in  $\mathcal{P}$  tends to drive the optimum towards lower cruising speed. In all the cases studied, turboprop aircraft show a lower  $\mathcal{P}$  compared to the pistonprop type.

## Nomenclature

$A_{pins}$	= passenger insurance amount, U.S. \$	$H$	= altitude, km
AR	= aspect ratio	$h_{oper}$	= duration of flight operations, h/day
$C_{aacq}$	= aircraft acquisition cost	$h_{maint}$	= aircraft maintenance manhour/flight h
$C_{ac}$	= aircraft related costs	$K$	= segment comfort factor
$C_{adep}$	= aircraft depreciation cost	$L_{rw}$	= runway length, km
$C_{ains}$	= aircraft insurance cost	$L/D$	= lift-to-drag ratio
$C_{aint}$	= aircraft interest cost	$N_{cap}$	= aircraft passenger capacity
$C_{crew}$	= crew cost	$N_{con}$	= number of nodes connected to each node
$C_D$	= drag coefficient	$N_{crew}$	= aircraft crew capacity
$C_{fuel}$	= fuel cost	$N_{pax}$	= daily passenger load on the network
$C_L$	= lift coefficient	$N_{rw}$	= number of nodes on the network
$C_{maint}$	= aircraft maintenance cost	$\mathcal{P}$	= penalty, U.S. \$/passenger
$C_{pins}$	= passenger insurance cost	$P1 \dots P4$	= pistonprop aircraft cases
$C_{rdep}$	= runway depreciation cost	$p/W$	= aircraft power to weight ratio, kW/kg
$C_{rint}$	= runway interest cost	$Q$	= fleet size
$C_{rw}$	= runway related costs	$R_{ains}$	= annual aircraft insurance rate
$C_{const}$	= runway construction cost, U.S. \$/km	$R_{crew}$	= flight crew remuneration rate, U.S. \$/block h
$c_{la}$	= runway land acquisition cost factor	$R_{fuel}$	= fuel price, U.S. \$/kg
$c_{land}$	= airport landing charges per trip	$R_{int}$	= annual interest rate on invested capital
$D_{ac}$	= aircraft depreciation period, yr	$R_{maint}$	= maintenance personnel remuneration rate, U.S. \$/maintenance manhour
$D_{rw}$	= runway depreciation period, yr	$R_{pins}$	= annual passenger insurance rate
$F$	= fare for each segment, U.S. \$	$S$	= area, m <sup>2</sup>
$\mathcal{F}$	= fare for the flight segment, U.S. \$	$T1 \dots T4$	= turboprop aircraft cases
$f$	= fare per distance, segments 1 and 5, U.S. \$/km	$t$	= time, h
		$t/c$	= thickness ratio
		$U_{annual}$	= annual aircraft utilization, h/yr
		$U_{Ctype}$	= undercarriage type
		$V$	= velocity, km/h
		$\mathcal{V}$	= passengers' value of time, U.S. \$/h
		$W$	= weight, kg
		$W/S$	= wing loading, kg/m <sup>2</sup>
		$X$	= internode range, km
		$x$	= segment length, km
		$x_{opt}$	= optimum runway location from network node, km
		$\beta$	= passenger load factor
		$\eta_p$	= propulsive efficiency
		$\omega_{oper}$	= frequency of aircraft operation, /h

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## Subscripts

blk	= block
cr	= cruise
$f$	= fuel
$i$	= segment
max	= maximum
$w$	= wing
0	= gross

## I. Introduction

A PASSENGER commuting by any means of public transportation system has to pay a fare, which is the sum of direct and indirect operating costs together with a profit for the operator. Apart from this, a passenger also spends indirectly in terms of time spent in traveling, hence, its money equivalent should also be added to the fare to obtain the net amount spent in travel. From the point of view of the passenger, the operator should in some way minimize the net amount spent, and not just the part that contributes to the fare alone.

The amount of the indirect spending depends on the standard of living and the importance the passenger assigns to his time. This factor can be taken care by defining the term value of time  $\mathcal{V}$ , which represents the money equivalent of each unit of the passenger's time.  $\mathcal{V}$  takes care of the commonly observed fact that the passengers are usually willing to pay some extra amount if the journey is completed in a lesser time.

The (negative) effect of this indirect spending could be reduced if the passenger could gainfully utilize the time spent during the course of the travel. In the context of long distance air travel, for instance, a business traveler may not mind paying an extra fare if facilities like a telephone and telefax are provided aboard the aircraft. On the other hand, uncomfortable journeys, in fact, lead to an apparent increase in the indirect spending! This aspect could be considered by dividing the time spent in travel by a comfort factor  $K$ , with unity as the default value and values increasing in proportion with the perceived passenger comfort and utility. (Conversely, values lesser than unity would correspond to the extent of passenger discomfort.)

Apart from the actual journey by the mode of public transport, a travel plan usually consists of other segments such as commuting from starting point to public transport terminus (i.e., coach station, railway station, airport, etc.), and from the second public transport terminus to the destination. Each of these segments will contribute to the (direct and indirect) spending, depending on the distance to be traveled, the means of transportation (with a corresponding comfort factor  $K_i$  associated with it), and the speed of the vehicle. Further, the waiting time at the end of some segments (which may be inevitable), contributes to the indirect spending, and could be quite significant if the passengers'  $\mathcal{V}$  is high. This could be accounted for by regarding them as extra segments of the travel plan with zero fare. A generalized penalty function  $\mathcal{P}$  for a travel plan consisting of  $n$  segments can thus be considered as

$$\mathcal{P} = \sum_{i=1}^n \left[ F_i + \frac{t_i \cdot \mathcal{V}}{K_i} \right] \quad (1)$$

Minimization of  $\mathcal{P}$  could be helpful in the selection of the best vehicle for a particular network from the ones currently available, from the point of view of the passengers. Further, if a new vehicle is to be designed to meet the needs of a specific network (e.g., high speed supersonic transport aircraft or a new generation mass transport vehicle),  $\mathcal{P}$  could be fruitfully integrated in the conceptual design and optimization studies. Many methodologies have been proposed for the conceptual design and optimization of transport aircraft. Of these, OPDOT,<sup>1</sup> and related methodologies developed by Arbuckle

and Sliwa,<sup>2</sup> and Sliwa,<sup>3,4</sup> CPDS developed by Wallace,<sup>5</sup> studies by Dovi and Wrenn,<sup>6</sup> and Simos and Jenkinson<sup>7</sup> are noteworthy. However, none of these are suitable to integrate  $\mathcal{P}$  directly in the conceptual design process, hence, a new methodology was developed.<sup>8</sup> This article describes this methodology, and how it can be employed for civil aircraft conceptual design optimization.

## II. Conceptual Design of an Aircraft for Air-Taxi Operations

Let us see how the methodology outlined in Ref. 8 can be applied to the conceptual design optimization of a general aviation/transport category aircraft. The aircraft is to be employed for air-taxi operations over short distances. The aim is to arrive at the optimum value of a few aircraft parameters (e.g., passenger capacity, wing loading, type of landing gear, etc.), and operating parameters (cruise speed, cruise altitude, etc.), that result in the minimum  $\mathcal{P}$  for operation over a specific network. Only two types of powerplants, viz., reciprocating and turboprop are considered, and twin-engine configurations of both these types are optimized, using "rubber-engine" sizing methods. The methodology is targeted at virgin networks, i.e., for those cities that are not already connected by air, hence, is applied over a hypothetical network. However, it could be modified to apply to existing networks too.

Let us consider a hypothetical network connecting  $N_{rw}$  airports spaced  $X$  apart from each other, with a total of  $N_{\text{pass}}$  passengers (each having an average value of time  $\mathcal{V}$ ) traveling per day over the network. A part of such a network consisting of seven contiguous cities is shown in Fig. 1. Passengers would actually be commencing (and ending) their journey from various points within the city, but on an average, it can be assumed that all journeys start and end from one location within each city (e.g., the city center); each such point is called a node of the network. It is further assumed that the passenger load is evenly distributed over the network, i.e., equal number of passengers commence their journey from each node to all (and only one of) the neighboring nodes. A typical travel plan from node A to node B of such a network is comprised of the following five segments (refer to Fig. 1), each contributing to the  $\mathcal{P}$  defined in Eq. (1): 1) journey from node A to the first airport, 2) waiting at the first airport, 3) the actual flight segment, 4) waiting at the second airport, and 5) journey from the second airport to node B.

The equation for  $\mathcal{P}$  for this travel plan (with terms for each segment listed separately) can be obtained as follows:

$$\begin{aligned} \mathcal{P} = & x_1 \cdot f_1 + \mathcal{V} \cdot \frac{x_1}{V_1 \cdot K_1} \quad (\text{segment 1}) \\ & + \mathcal{V} \cdot \frac{t_2}{K_2} \quad (\text{segment 2}) \\ & + \mathcal{F} + \mathcal{V} \cdot \frac{X}{V_{\text{blk}} \cdot K_3} \quad (\text{segment 3}) \\ & + \mathcal{V} \cdot \frac{t_4}{K_4} \quad (\text{segment 4}) \\ & + x_5 \cdot f_5 + \mathcal{V} \cdot \frac{x_5}{V_5 \cdot K_5} \quad (\text{segment 5}) \end{aligned}$$

Combining the terms involving  $\mathcal{V}$ , the previous equation becomes

$$\mathcal{P} = x_1 \cdot f_1 + x_5 \cdot f_5 + \mathcal{F} + \mathcal{V} \cdot \left( \frac{x_1}{V_1 \cdot K_1} + \frac{t_2}{K_2} + \frac{X}{V_{\text{blk}} \cdot K_3} + \frac{t_4}{K_4} + \frac{x_5}{V_5 \cdot K_5} \right) \quad (2)$$

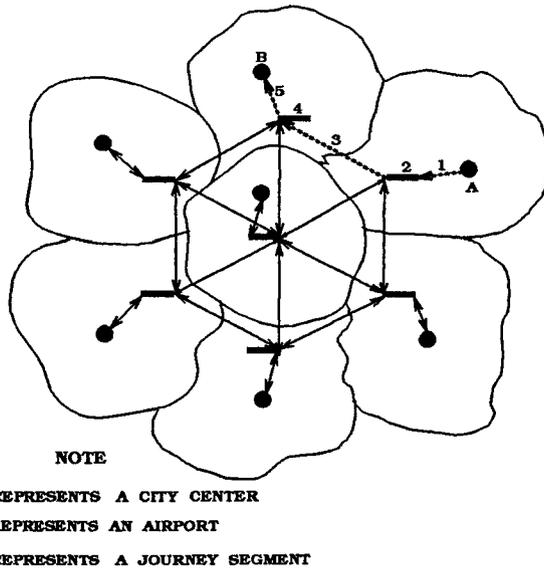


Fig. 1 Typical network of seven cities.

There are many unique features of the methodology proposed in Ref. 8, which are worth highlighting here. The optimum runway length required for operating over the network is determined, so that a rough estimate of the cost of constructing and maintaining a runway in each airport of the network is made. The location of the airport from the city center becomes an important parameter governing the runway cost, hence, its optimum value is also determined. A provision has been made, if need be, to recover the runway construction cost from the passengers using it during its operational life, as a part of their fare for the flight segment  $\mathcal{F}$ . The effect of variation of many parameters that are usually considered as constants in the conventional approach (such as cruising speed  $V_{cr}$  and aircraft capacity  $N_{cap}$ ) is considered, enabling their effect on fleet size, frequency of operation, and hence, on  $\mathcal{F}$  and  $\mathcal{P}$  to be incorporated. For example, the frequency of operation between any two adjacent nodes is

$$\omega_{oper} = \frac{N_{pax}}{N_{rw} \cdot \beta \cdot (N_{cap} - N_{crew}) \cdot N_{con} \cdot h_{oper}} \quad (3)$$

At the first node, the passenger would probably be in a position to minimize the waiting time  $t_2$  to some extent, by planning the journey in accordance with the flight schedules of the operator, but the same is not true about  $t_4$ , which is directly linked to  $\omega_{oper}$ . Assuming that the passengers wish to reach their destination in equally spaced intervals, some passengers would reach their destination just in time, while some others would reach a bit too early (since they would have been just a little late if they had taken the next flight). Thus, the average waiting time could be considered as half the time between two flights, i.e., half of  $\omega_{oper}$ . Hence,

$$t_4 = 1/(2 \cdot \omega_{oper}) \quad (4)$$

Similarly, the fleet size required to fulfill the needs of the network can be estimated as

$$Q = \frac{365 \cdot N_{pax} \cdot t_{blk}}{\beta \cdot (N_{cap} - N_{crew}) \cdot U_{annual}} \quad (5)$$

All parameters relevant to the methodology are classified as follows.

#### A. Network Variables

The four parameters that characterize the network, viz.,  $X$ ,  $N_{pax}$ ,  $N_{rw}$ , and  $\mathcal{V}$  are termed network variables, and are

Table 1 Values assigned to network variables

Symbol	Parameter	Value
$X$	Average internode range	216 (+108)
$N_{pax}$	Passenger load/day, entire network	12,000
$N_{rw}$	Number of runways	100
$\mathcal{V}$	Average value of time of each passenger	60

Table 2 Constraints and step size for design variables

Symbol	Parameter	Constraints	Step
$N_{cap}$	Aircraft capacity	$3 \leq N_{cap} \leq 60$	1
$V_{cr}$	Cruising speed	$\leq 630$	1.0
$W/S$	Wing loading	NIL	2.44
$C_{L,max}$	Maximum lift coefficient	$\leq 3.0$	0.1
$(AR)_w$	Wing aspect ratio	$\geq 5.00$	0.1
$(t/c)_w$	Wing thickness ratio	$\geq 10$	1
$H_{cr}$	Cruising altitude	$0.46 \leq H_{cr} \leq 3.0$	100
$U_{Ctype}$	Landing gear type	Fixed/retractable	—

not altered during an optimization cycle. It is assumed that their values are available as inputs by a market survey and/or other sources. Table 1 lists the values assigned to the network variables.

#### B. Design Variables

The independent variables that characterize the aircraft (and the way it's operated) are termed as design variables and their effect on  $\mathcal{F}$  and  $\mathcal{P}$  is investigated. Table 2 lists the eight design variables, along with the constraints imposed on their values, and the (minimum) step sizes used in the optimization. The constraints are imposed mainly from the point of view of validity of certain empirical equations used, and based on experience. For instance, the cruise altitude is limited to 0.46 km on the lower side at the end of the final segment of climb.<sup>9</sup> The upper limit is taken to be 3 km as unpressurized cabins are assumed. Although the runway length required for safe operation is an important variable, it does not appear in Table 2, but is indirectly considered as a function of  $W/S$  and  $C_{L,max}$ .

#### C. Design Constants

The parameters whose effect on  $\mathcal{P}$  is not considered are termed design constants, to keep the number of design variables within a manageable limit. The designer has no control over most of these, e.g., man-hour rates of activities such as tooling, manufacturing, etc. It was felt that for the present problem,  $\mathcal{P}$  would anyway be a very weak function of most of these variables (e.g., wing quarter-chord sweep, thickness-and taper-ratio of the tails), justifying their exclusion. A detailed list of these design constants and the values assigned to them is given in Ref. 8.

#### D. Initiators

Initiators are the five parameters, viz.,  $W_0$ ,  $\eta_p$ ,  $V_{max}$ ,  $(L/D)_{cr}$ , and  $p/W$  that have to be assigned some initial values, which automatically get corrected as the iterations proceed.

Figure 2 explains the above classification used for the various parameters.

### III. Estimation of Costs and $\mathcal{F}$

A number of techniques for the calculation of direct and indirect operating costs are reported in literature.<sup>10,11</sup> Again, none of these could be directly applied to enable estimation of the total costs in terms of the network and design variables, hence, new formulas are developed, as described below. The

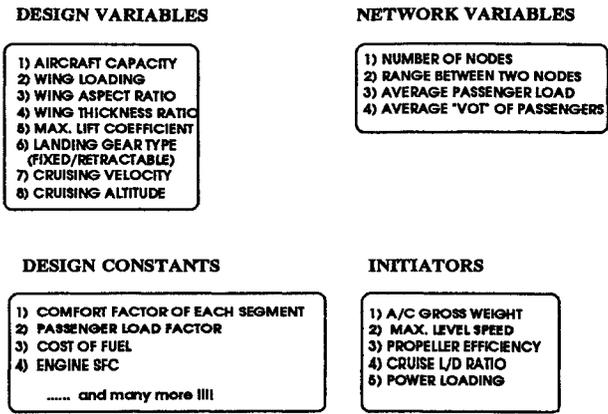


Fig. 2 Classification of parameters.

operating costs are considered as aircraft-independent and aircraft-dependent. The aircraft-independent costs (which include costs towards air traffic-control, security checks, baggage handling, ticketing, advertising, refreshments, administrative costs, etc.), have not been included in  $\mathcal{F}$ , since they are deemed to be almost invariant with the design variables. The aircraft-dependent costs are classified as aircraft related costs  $C_{ac}$ , and runway related costs  $C_{rw}$ . Hence,  $\mathcal{F}$  is estimated as

$$\mathcal{F} = C_{ac} + C_{rw} \quad (6)$$

It may be noted that the profit that the operator would charge as a part of  $\mathcal{F}$  has not been included in Eq. (6), since it varies from operator to operator, and is not a function of the design variables. In the equations that follow, all the cost terms have been calculated in terms of currency units (U.S. \$ 1986 base) per trip per passenger.

**A. Aircraft Related Costs**

$C_{ac}$  is estimated as the summation of the costs towards depreciation of and interest payable on the aircraft first cost, aircraft and passenger insurance, fuel consumption, crew salaries, and aircraft maintenance.

**1. Aircraft Depreciation Cost**

Depreciation is the allocation of the aircraft initial cost over the operating life of the aircraft and is estimated as

$$C_{adep} = \frac{C_{aacq} \cdot X}{\beta \cdot (N_{cap} - N_{crew}) \cdot U_{annual} \cdot D_{ac} \cdot V_{blk}} \quad (7)$$

**2. Aircraft Interest Cost**

This cost relates to the repayment of the interest on the capital invested for aircraft procurement. The book value of the aircraft<sup>11</sup> is the value at any point of time during the depreciation period. The average book value of the aircraft is considered for calculation of interest, assuming zero residual cost at the end of the depreciation period. Hence,

$$C_{aint} = \frac{C_{aacq} \cdot R_{int} \cdot X}{2 \cdot (N_{cap} - N_{crew}) \cdot U_{annual} \cdot V_{blk}} \quad (8)$$

**3. Aircraft Insurance Cost**

During its operational life, the aircraft is to be insured, and on the same lines as Eq. (8), this cost is estimated as

$$C_{ains} = \frac{C_{aacq} \cdot R_{ains} \cdot X}{2 \cdot (N_{cap} - N_{crew}) \cdot U_{annual} \cdot V_{blk}} \quad (9)$$

**4. Passenger Insurance Cost**

The costs associated with the insurance of the passengers is estimated as

$$C_{pins} = \frac{A_{pins} \cdot R_{pins} \cdot X}{U_{annual} \cdot V_{blk}} \quad (10)$$

**5. Crew Cost**

The crew cost includes the salary of the pilots and the cabin crew, and is estimated as

$$C_{crew} = \frac{R_{crew} \cdot t_{blk}}{\beta \cdot (N_{cap} - N_{crew})} \quad (11)$$

**6. Fuel Cost**

The fuel cost is estimated as

$$C_{fuel} = \frac{W_{fblk} \cdot R_f}{\beta \cdot (N_{cap} - N_{crew})} \quad (12)$$

**7. Aircraft Maintenance Cost**

The cost towards aircraft maintenance is estimated as

$$C_{maint} = \frac{h_{maint} \cdot t_{blk} \cdot R_{maint}}{\beta \cdot (N_{cap} - N_{crew})} \quad (13)$$

The cost towards aircraft spares is not considered due to nonavailability of relevant data. Hence,

$$C_{ac} = C_{adep} + C_{aint} + C_{ains} + C_{pins} + C_{crew} + C_{fuel} + C_{maint} \quad (14)$$

**B. Runway Related Costs**

In the estimation of  $C_{rw}$ , the costs associated with the construction of the new runway are estimated. These (together with the interest to be paid on this initial investment) are then charged to the passengers as a part of  $\mathcal{F}$ , if need be. The costs associated with the runway maintenance are indirectly considered by assuming its depreciation period  $D_{rw}$  to be small (15 years).

**1. Cost of Runway Construction**

The cost associated with the runway construction can be estimated as

$$C_{rc} = [(c_{la}/S) + c_{const}] \cdot L_{rw} \quad (15)$$

$c_{la}$  takes care of the (usually observed) fact that farther the runway is built from the city center, lesser are the costs associated with the land acquisition. Equation (15) neglects the effect of increase in width and strength of the runway due to aircraft size (and wheel track). It is also not valid for an airport located at or very close to the city center. In any case, it is very unlikely that enough space to build a runway at or near the city center would be available.

**2. Cost of Runway Depreciation**

The total cost of constructing runways at each node is divided equally among all the passengers who transit over the network in its depreciation period. Thus,

$$C_{rdep} = \frac{C_{rc} \cdot N_{rw}}{N_{pax} \cdot D_{rw} \cdot 365} \quad (16)$$

**3. Runway Interest Cost**

On the same lines as the aircraft interest cost, this cost relates to the interest payable on capital invested on runway construction. Here again, the average book value of the runway is considered, assuming zero residual value at the end of

the depreciation period, and the amount is charged to each passenger traveling over the network. Hence,

$$C_{rint} = \frac{C_{rc} \cdot N_{rw} \cdot R_{int}}{2 \cdot N_{pax}} \quad (17)$$

The runway related costs could thus be summed up as

$$C_{rw} = C_{rdep} + C_{rint} \quad (18)$$

If the methodology is to be applied to a network of cities with runways already existing, then Eq. (18) could be replaced by the following:

$$C_{rw} = \frac{c_{land}}{\beta \cdot (N_{cap} - N_{crew})}$$

### C. Estimation of $x_{opt}$

From Eq. (2), it can be seen that the location of the runway from the city center would affect  $x_1$  and  $x_5$ , and, hence, the penalty of these segments. The sum of the terms that depend on the runway location is given by the following expression:

$$C_{rdep} + C_{rint} + x_1 \cdot f_1 + x_5 \cdot f_5 + \mathcal{V} \cdot \left( \frac{x_1}{V_1 \cdot K_1} + \frac{x_5}{V_5 \cdot K_5} \right)$$

$x_{opt}$  for a given  $\mathcal{V}$  and  $L_{rw}$  is obtained by assuming  $x_1 = x_5 = S$ , substituting the expressions for  $C_{rdep}$  and  $C_{rint}$  [from Eqs. (16) and (17)] in the preceding expression, differentiating with respect to  $S$ , and equating it to zero as

$$x_{opt} = \frac{\left[ \left( \frac{N_{rw} \cdot L_{rw} \cdot c_{la}}{N_{pax} \cdot 365} \right) \cdot \left( \frac{R_{int}}{2} + \frac{1}{D_{rw}} \right) \right]^{0.5}}{f_1 + f_5 + \mathcal{V} \cdot \left( \frac{1}{V_1 \cdot K_1} + \frac{1}{V_5 \cdot K_5} \right)} \quad (19)$$

If the first and the last segments of travel are by the same mode of transport (as is usually the case), then Eq. (19) reduces to

$$x_{opt} = \frac{\left[ \left( \frac{N_{rw} \cdot L_{rw} \cdot c_{la}}{N_{pax} \cdot 365} \right) \cdot \left( \frac{R_{int}}{2} + \frac{1}{D_{rw}} \right) \right]^{0.5}}{2 \cdot \left( f + \frac{\mathcal{V}}{V \cdot K} \right)} \quad (20)$$

## IV. Algorithm for Conceptual Design

An algorithm was developed in Ref. 8 to incorporate  $\mathcal{P}$  in the procedure for conceptual design of an air-taxi aircraft. It consists of the inner (calculation) loop and the outer (optimization) loop. In the inner loop,  $\mathcal{P}$  is estimated for a set of design variables and the network parameters. In the outer loop, the set of design variables is judiciously varied, to arrive at the one that leads to the minimum  $\mathcal{P}$ , within a user specified tolerance limit.

### A. Estimation of $\mathcal{P}$ for One Set of Design Variables

This loop consists of modules for gross weight estimation, drag estimation, performance estimation, and cost estimation taken from Refs. 9 and 12, as shown in Fig. 3. The values of the initiators are corrected in an iterative manner through the relevant modules. Suitable tolerances for these corrections were determined by trial and error, which were  $\pm 0.5$  for  $W_0$ ,  $\pm 10^{-2}$  for  $V_{max}$ ,  $\pm 10^{-4}$  for  $(L/D)_{cr}$ , and  $\pm 10^{-4}$  for  $p/W$ .  $\eta_p$  is corrected in the module for  $W_0$  estimation within  $\pm 10^{-3}$ . The details of this loop are given in Ref 8.

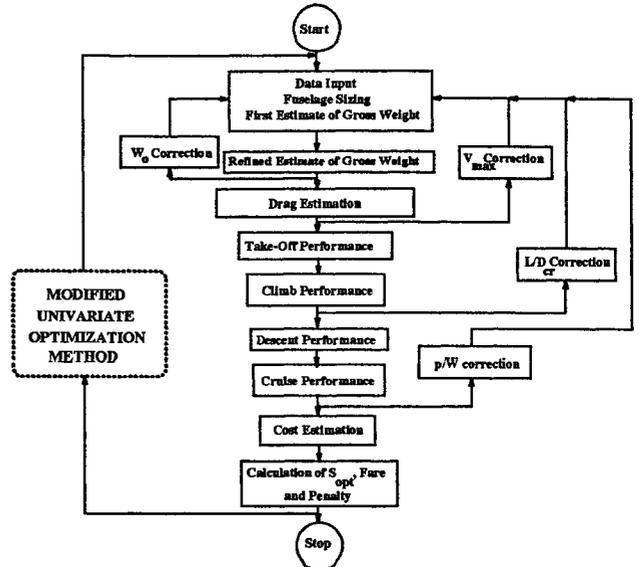


Fig. 3 Conceptual design process.

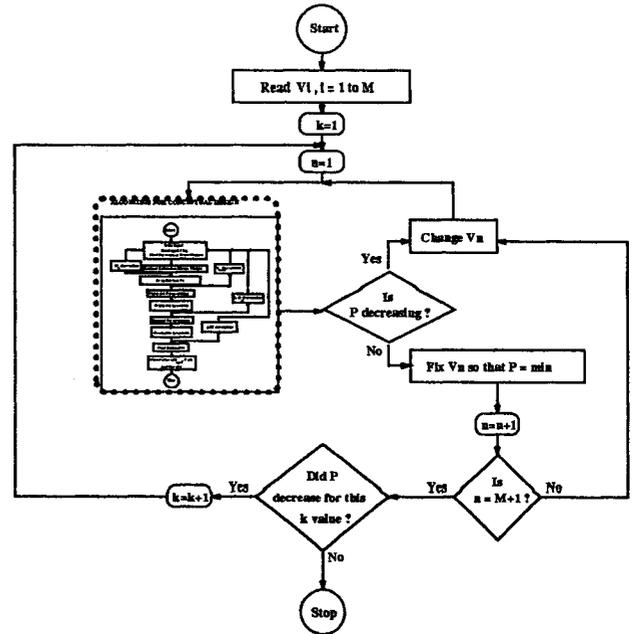


Fig. 4 Modified univariate optimization method.

### B. Optimization Procedure (Outer Loop)

The optimization procedure resembles the univariate method,<sup>13</sup> wherein the design variables are optimized one at a time, keeping others constant, as shown in Fig. 4. The interactive nature of this optimization procedure enables a substantial reduction in the number of calculation cycles required, by closely monitoring the trend of  $\mathcal{P}$  with change in the values of design variables. For instance, larger step sizes for design variables are used in the initial stages, which are then reduced in each cycle as the optimum is approached. The step sizes listed in Table 2 are the minimum values.

On an Intel 80486-based personal computer, the inner loop needs about 5 s CPU time, on an average. For some combinations of design variables, however, the algorithm does not converge; these cases are abandoned as infeasible after a user-specified number of iterations (say 200) are exceeded. The number of cycles needed for the outer loop varies, and greatly decreases with experience. On average, the entire exercise can be completed in about 1 h, most of which is

needed to decide and input the set of values of design variables for each calculation of the inner loop, since this is done manually. To confirm that this method leads to the global (and not just a local) optimum set of design variables, it was tried out for several different initial sets of design variables; it was found that the same optimum was reached, within limits of convergence.

### V. Results: Case Studies

In Ref. 8, eight case studies were carried out, four each for pistonprops and turboprops, as shown in Tables 3 and 4. Cases *P1* and *T1* result from the conventional approach to aircraft design, and can be considered as the baseline for each type, since  $\mathcal{V}$  and  $C_{rw}$  are not considered in  $\mathcal{P}$  estimation. The remaining three cases for each type arise when  $\mathcal{V}$  and/or  $C_{rw}$  are considered. The \* indicates that the values of the design variables are at the constraint boundaries, and would increase/decrease further if the constraints were relaxed. It was found that in cases *P1*, *T1*, *P3*, and *T3*, since  $C_{rw}$  was not considered,  $L_{rw}$  was approaching unrealistically high numbers (beyond 10 km or so), owing to the increasing  $W/S$  and decreasing  $C_{L_{max}}$  values. A constraint was therefore imposed in such cases that

$L_{rw}$  be governed by landing considerations rather than takeoff. It must be noted that in these cases, since  $C_{rw}$  has not been considered, the segments for city travel have also not been penalized.

Comparison between *P1* and *P2* (and *T1* and *T2*) reveals the effect of considering  $C_{rw}$ . It was found that, as expected, the takeoff considerations govern the optimum. There is a tendency to reduce  $L_{rw}$ , both directly by decrease in  $W/S$  and increase in  $C_{L_{max}}$  and indirectly by reduction in  $W_0$  [as seen from decrease in  $(AR)_w$  and increase in  $(t/c)_w$ ]. The reduced  $W/S$  causes the  $C_{L_{cr}}$  and  $V_{cr}$  to decrease. The increased  $S_w$  further causes  $C_{D_0}$  to decrease. On the other hand, cases *P1*, *T1*, *P3*, and *T3* reveal that if  $C_{rw}$  is not considered, cruise conditions drive the optimum.

A comparison between *P1* and *P3* (and *T1* and *T3*) reveals the effect of considering  $\mathcal{V}$  of passengers. One of the immediate implications is to reduce time in all segments, especially  $t_4$ , which depends on  $\omega_{oper}$  and, hence, indirectly on  $N_{cap}$ . Low  $N_{cap}$  would lead to larger  $Q$  (for the same  $N_{pax}$ ) resulting in a high  $\omega_{oper}$ , hence, low  $t_4$ . It is for this reason that  $N_{cap}$  decreases sharply in *P3* and *T3*. In order to increase  $V_{cr}$ ,  $U\text{Ctype}$  changes from fixed to retractable. Further,  $p/W$  increases and  $(t/c)_w$  decreases to meet the high  $V_{cr}$  requirement. As expected, cruise conditions govern  $W/S$ , which therefore, becomes extremely high owing to increased  $V_{cr}$ . The increased  $W/S$  causes  $L_{rw}$ ,  $C_{L_{max}}$ , and  $C_{L_{cr}}$  to increase. Since there is a tendency to takeoff at a speed that would be optimum from the point of view of climbing,  $(AR)_w$  increases to reduce induced drag.

In the case studies discussed so far, the effect of  $C_{rw}$  and  $\mathcal{V}$  on the optimum was considered in isolation. If these two are now considered together, the result will be additive for those design variables that are driven in the same direction by the two taken individually. This expectation is substantiated by increase in  $C_{L_{max}}$  in cases *P4* and *T4*. However, design variables, which show conflicting trends for  $C_{rw}$  and  $\mathcal{V}$  considered in isolation, could now be governed by either of the two [e.g.,  $(t/c)_w$  and  $(AR)_w$ ].

By comparing the set of optimum design variables, it is found that the optimum configuration of *P1* (and *T1*) differs greatly from that of *P4* (and *T4*).  $\mathcal{F}$  and  $\mathcal{P}$  of *P1* (and *T1*) are much lower than the corresponding values for *P4* (and *T4*), which may lead to a wrong impression that *P1* (and *T1*) is a much better aircraft compared to *P4* (and *T4*). However, it may be noted that each of these cases has been optimized individually, and no other combination of design variables would result in a lower  $\mathcal{P}$  for each case. For example, aircraft *P1* would have  $\mathcal{P}$  of 161.95 if  $\mathcal{V}$  and  $C_{rw}$  were included (compared to  $\mathcal{P}$  of 98.69 for aircraft *P4*). Similarly, aircraft *T1* would have  $\mathcal{P}$  of 155.47 if  $\mathcal{V}$  and  $C_{rw}$  were included (compared to  $\mathcal{P}$  of 92.66 for aircraft *T4*).

In all the cases studied, the turboprop aircraft consistently show a lower  $\mathcal{P}$  than the pistonprops, despite the considerably higher cost of turboprop engines for the same power (U.S. \$75/hp for pistonprops<sup>14</sup> and U.S. \$500/hp for turboprops<sup>15</sup>). A few case studies were carried out for much lower values of  $U_{annual}$ , yet the turboprops showed lower  $\mathcal{P}$  compared to the pistonprops, indicating that for this network, they would be a better choice compared to the pistonprops. However, it may be kept in mind that the numerical values of the results (and the conclusions derived from them) are valid for the specific values of the network variables and the design constants only.

After obtaining optimum combination of design variables, their sensitivity on  $\mathcal{P}$  was investigated. For this,  $\mathcal{P}$  was calculated for a few off-optimum values of one of the design variables for case *T4*, while assigning optimum values for the remaining. This was repeated for all design variables (except  $U\text{Ctype}$ ). It was observed that  $\mathcal{P}$  is most strongly affected by  $N_{cap}$ , and that after the minima at six seats,  $\mathcal{P}$  steeply increases with increase in  $N_{cap}$ . This is because  $\omega_{oper}$  is inversely pro-

Table 3 Case studies for pistonprop aircraft

	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>
$\mathcal{V}$	0	0	60	60
$C_{rw}?$	No	Yes	No	Yes
$N_{cap}$	24	24	6	6
$Q$	74	79	246	291
$W/S$	219.62	104.93	551.48	163.49
$S_w$	20.34	45.91	3.70	11.83
$W_0$	4467.4	4817.4	2043.0	1933.6
$p/W$	0.1431	0.1527	0.2749	0.2234
$V_{cr}$	382	360	500	432
$V_{max}$	436	404	567	483
$U\text{Ctype}$	FIX	FIX	RETR	RETR
$(AR)_w$	7.0	5.2	7.9	5.0*
$(t/c)_w$	15	18	11	10*
$H_{cr}$	0.46*	0.46*	0.46*	0.46*
$C_{L_{max}}$	1.2	2.6	1.6	3.0*
$C_{D_0}$	0.0247	0.0168	0.0542	0.0228
$C_{L_{cr}}$	0.3267	0.1754	0.477	0.1897
$L_{rw}$	1.313	0.528	2.204	0.606
$C_{rw}$	0.0	4.91	0.0	11.07
$\mathcal{F}$	16.04	22.57	27.60	38.79
$\mathcal{P}$	16.04	25.77	75.10	98.69

Table 4 Case studies for turboprop aircraft

	<i>T1</i>	<i>T2</i>	<i>T3</i>	<i>T4</i>
$\mathcal{V}$	0	0	60	60
$C_{rw}?$	No	Yes	No	Yes
$N_{cap}$	24	39	6	6
$Q$	71	47	225	262
$W/S$	263.54	141.53	668.61	153.73
$S_w$	13.71	41.97	2.13	9.29
$W_0$	3614.3	5939.5	1425.7	1427.4
$p/W$	0.1218	0.1221	0.3194	0.2614
$V_{cr}$	396	367	551	486
$V_{max}$	437	406	600	524
$U\text{Ctype}$	FIX	FIX	RETR	RETR
$(AR)_w$	6.4	5.0*	7.3	5.0*
$(t/c)_w$	17	21	10*	10*
$H_{cr}$	0.46*	0.46*	0.46*	0.46*
$C_{L_{max}}$	1.8	3.0*	2.4	3.0*
$C_{D_0}$	0.0304	0.0197	0.0831	0.0259
$C_{L_{cr}}$	0.3639	0.2273	0.4769	0.1410
$L_{rw}$	1.111	0.565	1.840	0.588
$C_{rw}$	0.0	5.14	0.0	10.87
$\mathcal{F}$	13.28	19.01	23.24	34.97
$\mathcal{P}$	13.28	22.32	69.22	92.66

portional to  $N_{\text{cap}}$ , resulting in a substantial increase in  $t_4$  as  $N_{\text{cap}}$  increases. Since  $\mathcal{V}$  has been considered, the increase in  $t_4$  directly adds to  $\mathcal{P}$ . It may be noted that if  $\mathcal{V}$  is not considered, the optimum  $N_{\text{cap}}$  is 39 (refer case T2 in Table 4).

$\mathcal{P}$  was seen to increase almost linearly with  $H_{\text{cr}}$ . This can be attributed to the time lost (and the horizontal distance covered) in climbing to a greater altitude and then descending from that altitude for landing, resulting in a substantial reduction in the actual cruise distance. In all the case studies, the optimum  $H_{\text{cr}}$  was at the lower limit, i.e., 0.46 km. This is because of the low value of  $X$  for this network. To confirm this, case study was carried out with  $X = 1000$  km (+ 500 km reserve fuel). It was found that the optimum  $H_{\text{cr}}$  increased to the upper limit of 3 km, which clearly brought out the dependence of optimum  $H_{\text{cr}}$  on  $X$ , as expected.

The sensitivity analysis also revealed that  $\mathcal{P}$  increased very slightly with increase in  $(t/c)_w$  and  $(AR)_w$ , and that the optimum was quite flat for  $V_{\text{cr}}$  and  $W/S$ .  $C_{L_{\text{max}}}$ , however, directly affects the  $L_{\text{rw}}$  and, therefore,  $C_{\text{rw}}$ , resulting in a substantial variation in  $\mathcal{P}$ . The details of the sensitivity analysis are given in Ref. 8.

## VI. Conclusions

It can be concluded that integration of costs and economic parameters directly (in the form of  $\mathcal{V}$ ) in aircraft conceptual design strongly affects the optimum configuration. From the case studies for the air-taxi aircraft, the following can be concluded:

1)  $N_{\text{cap}}$  and  $V_{\text{cr}}$  (hence  $U$  type) depend primarily on  $\mathcal{V}$  of passengers.

2) Incorporation of  $C_{\text{rw}}$  in  $\mathcal{F}$  (hence,  $\mathcal{P}$ ) tends to reduce  $V_{\text{cr}}$  and  $W/S$ .

3) Turboprop aircraft appear to be superior to pistonprop type, even when very low  $U_{\text{annual}}$  is assumed.

The sensitivity analysis of the design variables shows that  $\mathcal{P}$  is strongly affected by  $C_{L_{\text{max}}}$ ,  $N_{\text{cap}}$  and  $H_{\text{cr}}$ , while it's almost invariant with  $(AR)_w$  and  $(t/c)_w$ .

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